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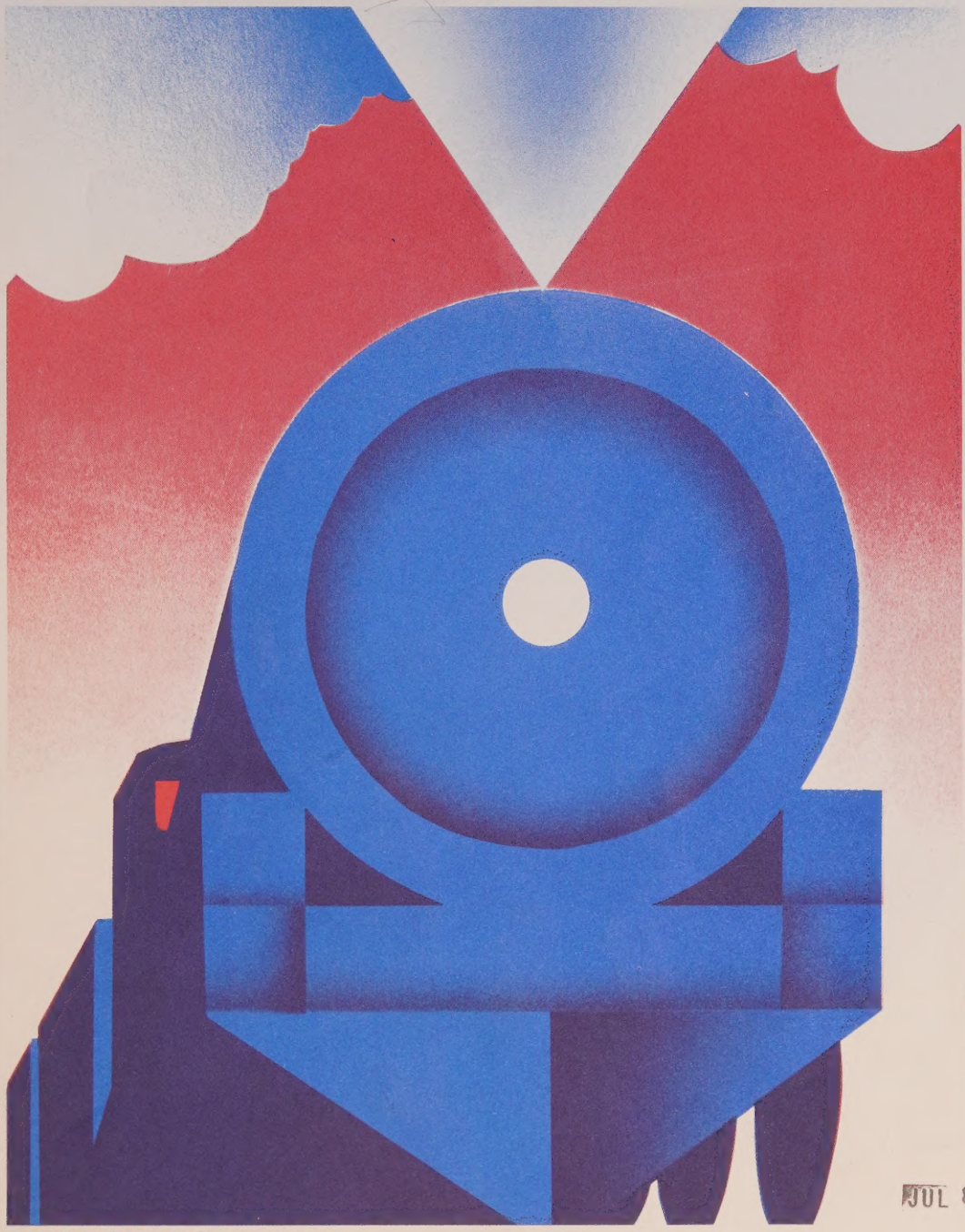
Curriculum Ideas for Teachers 1987

GEOMETRY Junior Division



CURRICULUM RESOURCES
OISE

JUL 8 1987

1. TEACHERS' NOTES

Ontario Ministry of Education
Hon. Sean Conway, Minister
Bernard J. Shapiro, Deputy Minister

Ontario

Ministry
of
Education

Ministère
de
l'Éducation

5-8948 17th Floor
Mowat Block
Queen's Park
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M7A 1L2

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Queen's Park
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MEMORANDUM TO: Directors of Education
Principals of Elementary Schools
Junior Division Teachers

FROM: Shannon A. Hogan

DATE: May, 1987

SUBJECT: Geometry, Junior Division

The Ministry of Education is pleased to make available the document Geometry, Junior Division which accompanies this memorandum. The French-language version of this publication will follow in September, 1987.

Geometry, Junior Division has been prepared to assist principals and teachers implement the learning opportunities identified for geometry in The Formative Years. The student activities and the accompanying notes in the document are intended to help teachers achieve these learning opportunities in group settings with the students actively involved in the learning process. Further, the activities are a basis for teachers and students to develop their own activities.

The Ministry of Education plans to introduce Geometry, Junior Division through invitational workshops. Information regarding these workshops will be provided by Ministry of Education Regional Offices.

Shannon A. Hogan
Director
Centre for Early Childhood
and Elementary Education

A Support Document to *The Formative Years*

GEOMETRY
Junior Division
1987

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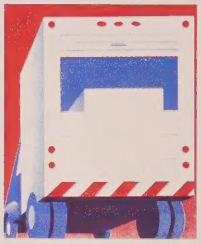

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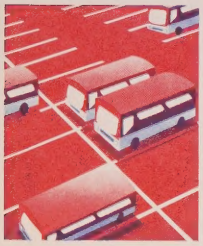

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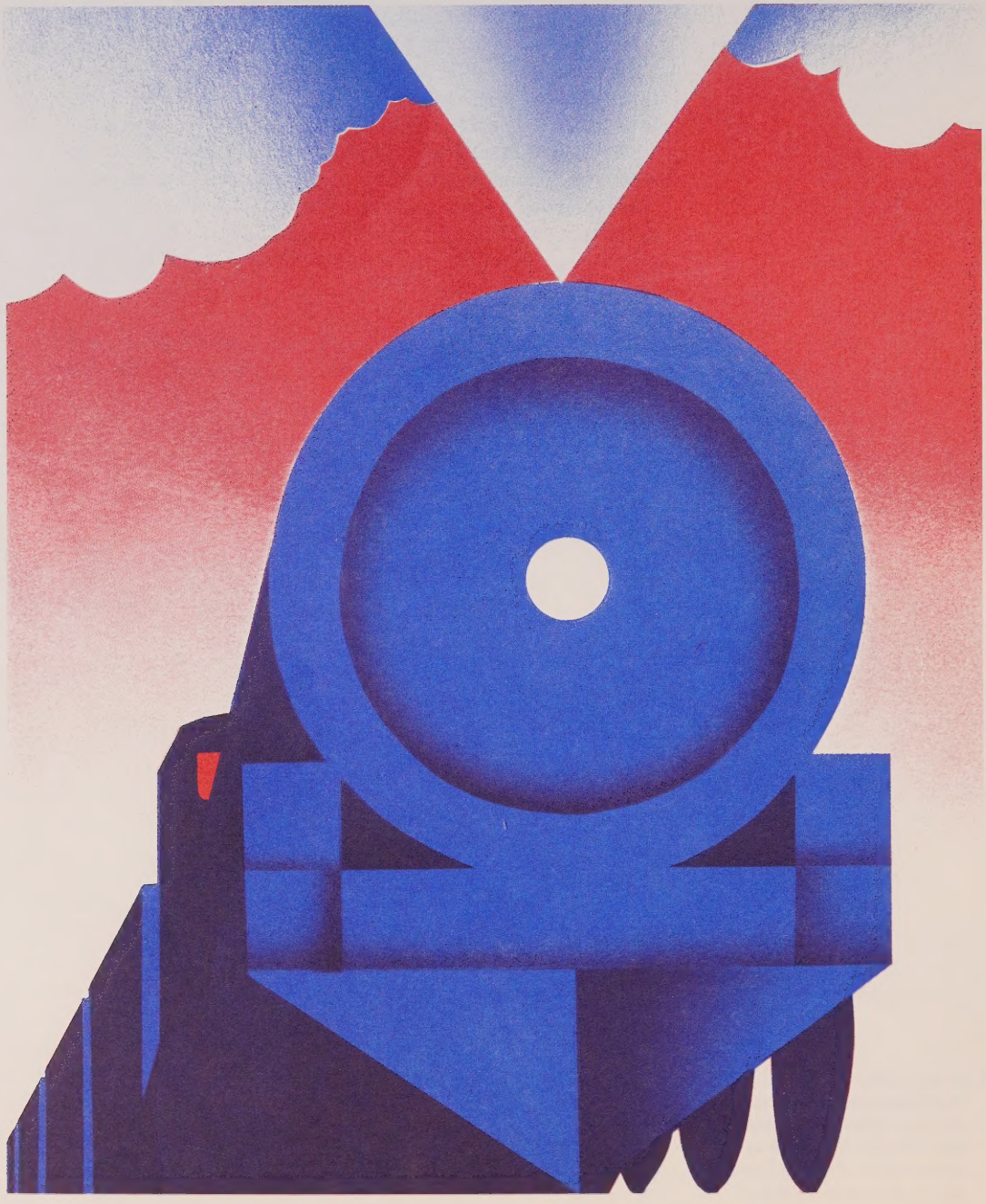
INTRODUCTION

Geometry, Junior Division is designed to assist teachers in achieving the aim for geometry set out in *The Formative Years* — that is, to help students understand increasingly complex spatial relationships.[1] Activities supporting the learning opportunities that accompany this aim are described in *Geometry, Junior Division.* They will enable teachers to establish a hands-on environment in which students can build on the learning they have experienced in the Primary Division. In keeping with the philosophy of education outlined in *Education in the Primary and Junior Divisions*, the activities described here will encourage students to investigate, think, appreciate, and create. In addition, these learning experiences will provide a basis for teachers and students to develop their own activities.

Geometry, Junior Division comprises three sections: *Teachers' Notes,* contained in this booklet, and *Learning Opportunities I-IX: Activities* and *Blackline Masters,* published as a separate booklet.

1. Ministry of Education, Ontario, *The Formative Years: Provincial Curriculum Policy for the Primary and Junior Divisions of the Public and Separate Schools of Ontario* (Toronto: Ministry of Education, 1975), p. 12.



GEOMETRY IN THE JUNIOR DIVISION

The Role of Geometry

Geometry is a basic topic in the field of mathematics and therefore deserves a place in the curriculum of the Junior Division. It is the "natural language of spatial concepts and space relationships in the real world. Space is a basic human experience with which every child of every age must come to terms".[2]

Geometry has important practical, scientific, and cultural applications. It plays a fundamental role in the physical sciences and engineering and some role in nearly every other subject or profession. The ability to visualize geometrically is a basic part of a scientist's mental equipment and an aid to everyone who wishes to understand and take part in the latest developments in technology. An understanding of geometry is important in the visual and kinetic arts and sciences and in many crafts and industries. In addition, geometry can provide a purely intellectual and aesthetic stimulus and pleasure.

Geometry may be described as the science of space and form, while arithmetic is the science of number. An understanding of spatial relationships is crucial if students are to develop the ability to quantify such relationships through measurement. At the pictorial representation stage, geometry plays a significant role in the development of students' understanding of and ability to use the four operations in arithmetic (for example, addition and subtraction can be represented by a one-dimensional number line, and multiplication and division by a two-dimensional rectangle). Both an understanding of the interrelationships of geometry, arithmetic, and measurement and a balance among them should be established in the Junior Division mathematics program. For students to develop the necessary thinking skills required for problem solving, all three of these aspects must receive adequate emphasis.

2. Ministry of Education, Ontario, *Education in the Primary and Junior Divisions* (Toronto: Ministry of Education, 1975), p. 74.

General Approach

Preschool geometrical experiences are of both three-dimensional shapes (e.g., boxes, balls, cans, cones, and blocks of all shapes and sizes) that can be felt as well as seen and two-dimensional shapes (e.g., pictures of familiar objects) that appeal primarily to the sense of sight. The very young child also uses manipulation to gain experience with objects in the environment and acquires insights based on observation of motion.

In school, the child at the Kindergarten-Primary level (JK-3) continues to need experiences with three-dimensional objects. Rudimentary organizing and classifying activities promote the child's readiness to deal with abstract mathematical notions, and the study of spatial relationships helps him or her to consolidate and extend an intuitive grasp of the properties of space and of objects in it. Through appropriate learning opportunities in the Primary Division, the child will have begun to "acquire basic and powerful concepts, such as those of pattern, symmetry, order, and similarity. . . . These general concepts are powerful because they can be applied to many different experiences in the arts, in movement, in music, in science, and in mathematics. They continue to develop through organized activities over a span of years."[3]

In the Junior Division, it remains important for students to be actively involved in exploring, examining, and manipulating three-dimensional and two-dimensional shapes. The students should be trying out their own ideas, making decisions, and drawing conclusions. They need to act on ideas themselves before they can assimilate information into their cognitive structures; experience is always necessary for intellectual development — a demonstration or explanation is not enough. Students should be encouraged to move from the observations they make while manipulating materials to the expression of mathematical ideas.

3. Ibid., p. 12.

Language

Language has a double role in the study of geometry. First, language is a vehicle for learning geometric concepts; second, the special vocabulary of geometry facilitates the expression of ideas about properties of space and relationships of objects in space.

New information, often obtained through exploratory experiences, fits and is linked with the known. The natural way for students to strengthen these links is by talking — by making the effort to put newly acquired knowledge into their own words. Verbalizing helps pupils to clarify, refine, and consolidate their learning.

Teaching should begin with language that has recognizable reference points for the children. Terms and definitions given to students without a meaningful context do not enhance learning and may hinder it. The mere recitation of terms and definitions often disguises the fact that no real grasp of the terminology or understanding of the concepts is being developed. The ability to use precise mathematical terms with understanding will develop through the process of exploratory talk during investigations and in follow-up discussions with peers and the teacher.

Students should develop a class, group, or individual glossary of geometric terms and definitions. It is advisable to use pictures and diagrams rather than verbal descriptions. Terms needed for a particular topic could be illustrated on a wall chart for ready reference.

Computers

Computers can be utilized effectively in the geometry program of the Junior Division. Curriculum-based software that allows for teacher input and is open-ended to encourage student investigation and exploration is becoming increasingly available. Such material includes programs to help students understand the concepts of symmetry and motions of slides, flips, and turns, among others. Computer programs should be used as a follow-up to "hands-on" activities.

Introduction

NOTE ON ORGANIZATION

As stated in the general introduction, this support document focuses on the aim of geometry and the accompanying learning opportunities outlined in *The Formative Years.* The nine learning opportunities provided in Junior Division geometry should enable students to:

— classify shapes in space (three-dimensional shapes) and in the plane (two-dimensional shapes) according to their attributes;
— discover patterns and identify properties of two-dimensional and three-dimensional figures by tiling in the plane and stacking in space;
— develop concepts of symmetry related to point, line, plane, rotational, and translational symmetry;
— identify the symmetries of figures in the plane and of objects in space;
— understand and use words such as *slide, turn,* and *flip* in describing motions of figures in the plane and in space;
— develop and use the concept of co-ordinates;
— understand, represent, and use concepts such as *vertical, horizontal, perpendicular, parallel, angle, triangle,* and *polygon;*
— understand the concept of congruency;
— identify relationships between similar figures and apply measurement skills in scale drawings.[4]

The body of this document is divided into three main sections, each of which deals with the nine learning opportunities in a different way:

— **The first section,** *Learning Opportunities,* is a discussion, addressed to the teacher, of the background, the materials, and the activities for each of the learning opportunities.
— **The second section,** *Learning Opportunities I-IX: Activities,* consists of instructions for activities, addressed to the students, for each of the learning opportunities. Each activity instruction sheet is marked with one, two, or three asterisks to indicate a level of difficulty. (The activities that are considered to be easiest are marked with one asterisk, those that are considered most challenging are marked with three.) When assigning activities, teachers should take each student's level of development into consideration, to ensure that students are not forced to memorize as a substitute for comprehension.
— **The third section,** *Blackline Masters,* contains a set of sample pattern sheets for use in the activities provided for each learning opportunity.

4. Ministry of Education, Ontario, *The Formative Years,* p. 12.

GROUPING OF STUDENTS

Although the instructions on the activity sheets are addressed to the individual student, it is assumed that students will be working in groups or pairs for most or all of these activities. Sometimes the size of the group for the assignment is suggested on the activity sheet or in the comments to the teacher. Most frequently the size and nature of the group is left to the teacher, who is in the best position to know the needs of each of the students. A conscious effort, however, should be made by the teacher to involve all students in the various roles—leadership, reporting, recording, etc.—that they carry out while working in groups.

TIME REQUIREMENTS

Some of the activities will be finished within a class mathematics period. Other activities may take several days to complete and may include investigations that take place at home, or on the way to and from school. The timing of any particular activity should be flexible, since the teacher may gain ideas for further activities and challenges while observing student behaviour during an activity.

MATERIALS

Because the following materials should be available to students at all times, they are not mentioned in the *You need* section of the activity sheets:

- pencils (including coloured pencils or crayons)
- rulers (30 cm and metre sticks)
- erasers
- scissors
- paste or glue
- blank paper (large and small sheets)

Other materials, such as cardboard, Bristol board, coloured construction paper, onion-skin paper, folding paper, etc., are listed and specifically described in the *Materials* section for each learning opportunity and are identified on each activity sheet. Materials to be constructed by teachers and/or students, as well as commercial materials, are also listed and described in each *Materials* section.

Blackline masters (BLMs) are provided for such things as 2-D shapes, squared paper, dot paper, number planes, etc., and are placed in the accompanying booklet. The appropriate BLMs are described in the relevant learning opportunities. In the comments on the learning opportunity activities, the teacher's attention is drawn to the specific blackline masters and to any special materials that must be prepared or made available for an activity.

RECORDING AND REPORTING

Specific suggestions are made on most activity sheets for ways of recording and displaying the results of investigations and constructions. The Ontario Ministry of Education support document *Focus on Writing*[5] provides useful information on students' writing. Teachers should encourage students to discuss and agree on the best ways to record the results of their investigations.

Groups may present the results of their activities to the class, to other groups, or to the teacher. *Teachers should encourage the students to compile a class, group, or individual folder for geometry.*

THE ROLE OF THE TEACHER

The teacher's role is to plan and arrange situations that take advantage of the students' interests and to participate actively in order to stimulate group and/or class discussions. The teacher must be both knowledgeable about the mathematical ideas involved and alert for opportunities to intervene in order to trigger a new line of thought or direct random thoughts towards a solution.

To use the geometry activities effectively, teachers should:
— choose appropriate activities to meet the needs of the students;
— have available and ready all materials needed;
— group the students appropriately;
— be ready to mediate and to ask questions that will enable a group or individual to move to a higher level of response or to find a way out of a blind alley;
— join in follow-up discussions to ask questions, encourage students to question each other, correct misinformation and errors in graphs, diagrams, or written reports, and encourage students to extend an activity or investigation;
— set an example by asking higher-order questions and encouraging students to do the same;
— observe student behaviour during both the activity and the follow-up discussion in order to evaluate student progress;
— evaluate the recording or reporting of results.

EVALUATION

When evaluating student progress in an activity-oriented program, teachers will find it useful to:
— observe the student's performance in both individual and group work;
— observe the student's performance when applying the learning from one activity to another similar activity;
— observe the student's participation in a class or group discussion;
— interview the student individually;
— encourage group, peer, and student self-evaluation;
— use the Ontario Assessment Instrument Pool (OAIP) for the Junior Division.

5. Ministry of Education, Ontario, *Focus on Writing* (Toronto: Ministry of Education, 1982).

Learning Opportunity I

- CLASSIFY SHAPES IN SPACE (THREE-DIMENSIONAL SHAPES) AND IN THE PLANE (TWO-DIMENSIONAL SHAPES) ACCORDING TO THEIR ATTRIBUTES

BACKGROUND NOTES

Sorting and classifying skills form a solid basis for the logical thought processes that are necessary both in mathematics and in other areas of the curriculum.

In the Primary Division the child sorts and re-sorts objects using colour, texture, shape, thickness, size, or function as the common property. In the Junior Division the child undertakes more complex classification activities, involving two or more attributes, with three-dimensional and two-dimensional shapes. These experiences deepen and extend the student's intuitive understanding of the properties of these shapes.

MATERIALS

Check the individual activity sheets for the complete list of materials for each activity.

3-D models Have available in the classroom a set of large wooden or plastic geometric models containing a minimum of: a cone, a sphere, a hemisphere, a cylinder, a cube, prisms, and pyramids. If possible, obtain a set of building blocks as well, to add to the variety of shapes and provide several blocks of the same shape and size for stacking and building activities. In order to provide a sufficient number and variety of 3-D models to make the sorting and classifying activities both challenging and worthwhile, you may need to supplement the set of 3-D models by collecting such objects as:

— spheres (balls of all sizes — e.g., Ping-Pong, golf, tennis, beach, sponge, rubber, Styrofoam "snowballs", alleys, etc.)

— cylinders (cans, plastic food containers, tennis-ball containers, badminton-bird containers, flashlight batteries, checkers pieces, coins, unsharpened round-shafted pencils, etc.)

— cones (party hats, drinking cups, badminton birds, ice-cream cones, pine cones, etc.)

— prisms (rectangular boxes and cartons, sugar cubes, dice, triangular chocolate candy boxes, hexagonal boxes [J-Cloths boxes, pizza boxes], unsharpened hexagonal-shafted pencils, etc.)

— pyramids (tetrahedron dice [four faces], tetrahedron creamers [some restaurants have these], etc.)

You may also make cardboard 3-D models from the nets (2-D patterns for making 3-D models) on blackline masters IV-2 through IV-10 to add to the set. For larger models, increase the size of the 2-D shapes in the pattern. Spray-paint the models to make them more durable. (Leftover car enamel spray paint does an excellent job.) Emphasize safety if pupils help in this task.

Sorting hoops Make these from yarn, string, or strips of paper stapled together, or purchase hula hoops or collapsible plastic grouping circles from educational suppliers.

2-D pictures of 3-D models BLM I-1 provides pictures of some 3-D models. Supplement these with pictures of 3-D objects of various shapes from old magazines and catalogues.

2-D shapes Have a group of students prepare a set of 2-D shapes by cutting out or tracing on cardboard the following: the 2-D shapes on blackline masters I-2, I-3, I-4, I-6, and II-3; a few of the pattern-block shapes on BLMs II-5 through II-10, and on BLMs IV-1, VIII-2, and IX-2. If possible, laminate the cardboard before the students cut out the shapes. Plastic or heavy cardboard 2-D shapes are available from educational suppliers.

Geoboards You will find the 5 x 5 or 25-nail geoboard suitable for many activities. The 10 x 10 or 100-nail geoboard allows room for the construction of more and larger shapes and for investigations of symmetrical relations. The 100-nail geoboard is also very useful in the Junior Division arithmetic program — for base-ten numeration, decimals, fractions, and per cent. Some activities require a triangular geoboard, which has a variety of additional uses. Geoboards may be purchased through most educational suppliers; or you may make your own, using squares (approximately 20 cm x 20 cm) of one-centimetre plywood and short finishing or upholstery nails. Paint or stain the plywood if you wish, then place the geoboard pattern on the board, hammer the nails part way into the plywood, rip off the paper pattern, and the geoboard is ready to use. Patterns for geoboards are on teacher blackline masters (BLMTs) 1 through 3:

— BLMT 1: 5 x 5 25-nail square geoboard
— BLMT 2: 10 x 10 100-nail square geoboard
— BLMT 3: triangular geoboard

You will also need assorted sizes of coloured rubber bands.

Blank cards Plasticized blank playing cards, approximately 6 cm x 9 cm, are available through some educational suppliers, or through playing-card manufacturers. However, for most purposes, cards cut from Bristol or tag board are satisfactory.

Plasticine Several of the activities call for Plasticine or some other form of non-hardening modelling clay.

Ontario official road map For class sets of this map used in Learning Opportunity I-4 (LO I-4) write to:

Ontario Ministry of Transportation
and Communications
Map Office
1201 Wilson Avenue
Downsview, Ontario
M3M 1J8

COMMENTS ON ACTIVITIES

LO I-1 Use the questions on the activity sheet as the basis for a follow-up discussion. Allow time for each group to present the results of its investigations. Encourage students to use their charts, pictures, tape recordings, etc., in their presentations.

LO I-2 The students will need copies of BLM I-1. During the follow-up session, encourage the students to compare, question, and discuss their own and the other groups' charts.

LO I-3 The students will need copies of BLM I-1.

Each picture of a 3-D model on BLM I-1 is meant to represent any and all models of that kind of shape. For example, the actual 3-D model of a hexagonal prism might be a J-Cloths box, while the picture of a hexagonal prism on BLM I-1 might look more like a pizza box.

Have the students match the picture with the model by feeling faces, edges, and corner points. They should determine that the J-Cloths box matches the picture when they feel the two hexagonal and the six rectangular faces.

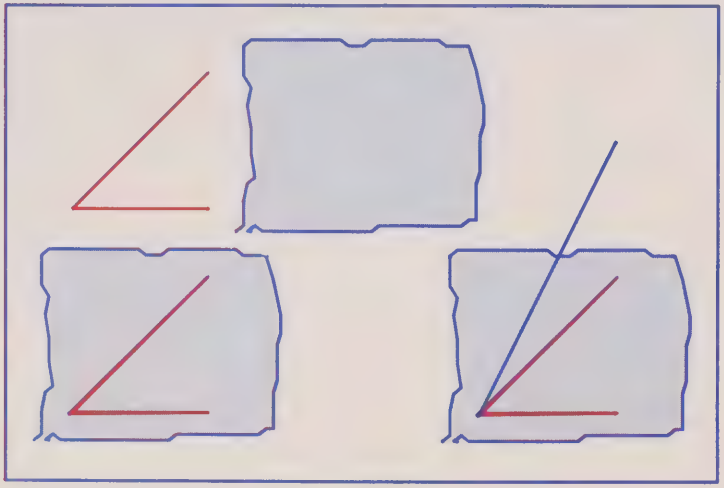
LO I-4 Before the follow-up session, make sure that the groups have had time to read the descriptions and examine the pictures organized by other groups. Have the models and some Plasticine balls on hand in case the need for a demonstration arises during the discussion.

LO I-5 Students will need copies of BLM I-1. The purpose of this activity is to reinforce the student's ability to associate a 3-D model with its geometric name.

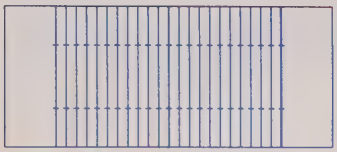
LO I-6 Some students may need the 2-D shapes on BLMs I-2, I-3, and I-4. For best results, have the students work in groups of three or four. Try to arrange with your principal to have the warning signs that the students create posted in appropriate places in the school.

LO I-7 The emphasis in this activity is on the general classification of polygons according to the number of sides. It is far more important for students to grasp this concept than for them to learn the special names given to each of these classes. They will come to know the geometric terms as a natural outcome of many activities with these shapes.

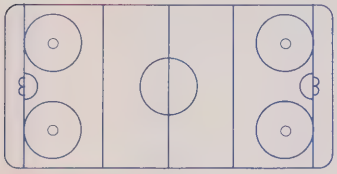
LO I-8 During the follow-up discussion, have the students develop a class list of the examples of right angles, acute angles, and obtuse angles they observed in their investigations. Show students how to compare angles by tracing and fitting. Use onion-skin paper to trace one of the angles to be compared, then move the tracing to place it on the other angle. Encourage the students to try to arrange the angle cards in order of size, using this method of comparison.



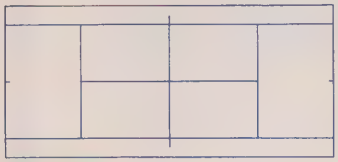
LO I-9 The students will need square dot paper (BLM I-5). Have the students work in pairs and report jointly on their experiences.



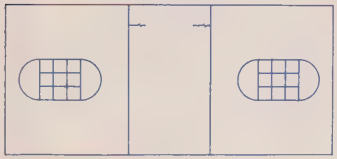
football field



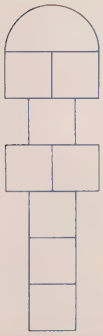
ice hockey rink



tennis court



shuffleboard



hopscotch layout

curling surface

LO I-10 The students will need a set of 2-D shapes. (See *Materials,* page 16.) Emphasize the concept of different types of triangles rather than the memorization of the names given to the different types. Help the students develop the ability to recognize different kinds of triangles by encouraging them to describe various triangles using their own words.

The students should come to realize that an obtuse-angle equilateral triangle cannot exist, and therefore cannot be traced. The other two types of triangles referred to in steps 3(a) and 3(b) should appear in two places.

LO I-11 A baseball diamond is a square; home plate is an irregular pentagon; the bases are squares; and the pitcher's rubber is a rectangle. In some parks the shaved portion of the grass around the rubber is a circle.

LO I-12 Have students work in groups of three or four. Each group will need copies of BLM I-6. If you are using plasticized blank playing cards, have the students trace the shapes onto the cards using a permanent felt marker.

Learning Opportunity II

- DISCOVER PATTERNS AND IDENTIFY PROPERTIES OF TWO-DIMENSIONAL AND THREE-DIMENSIONAL FIGURES BY TILING IN THE PLANE AND STACKING IN SPACE

BACKGROUND NOTES

Tiling in the plane and stacking in space are interesting and instructive ways of emphasizing many of the congruency, similarity, and angle properties of polygons and polyhedrons. In the Primary Division, through play activities with building blocks, pattern blocks, and attribute blocks, children have developed an intuitive awareness of these properties. In the Junior Division, as students gain experience in covering surfaces with two-dimensional shapes, they begin to identify properties of the plane figures that they are using. In a similar way, they discover properties of three-dimensional figures by attempting to make stacking patterns.

Tiling involves the use of two-dimensional shapes to cover a surface leaving no gaps or overlaps. Two-dimensional shapes will tile only if the angles meeting at the vertices of the pattern have a total measure of 360 degrees. Of all the regular polygons (that is, polygons with all sides and angles equal), only the equilateral triangle, the square, and the hexagon will tile.

The cube is the only *regular* polyhedron that will stack. Through play experiences with building blocks, very young children discover that rectangular prisms can be stacked. They may also find out that they can use other kinds of prisms in their buildings. After some experiences with three-dimensional models, students in the Junior Division will broaden and deepen their intuitive understanding of the properties of three-dimensional figures.

MATERIALS

Check the individual activity sheets for the complete list of materials for each activity.

3-D models See the *Materials* section of Learning Opportunity I (pages 15-16). The set of 3-D models used for LO II-1 must include building blocks.

Interlocking cubes Either 1 cm or 2 cm interlocking cubes may be used. For preference, choose cubes that interlock on five or six faces and come in a variety of colours. These can be purchased from most educational suppliers.

Tangram pieces The plastic or heavy cardboard tangrams available from educational suppliers may not be the correct size to fit on the shapes on LO II-5. You may cut tangram pieces that will fit from BLM II-4. To make the tangrams more durable, trace or paste them on Bristol board, laminate them if possible, and then cut them out.

Pattern blocks Purchase at least one set of pattern blocks from an educational supplier, if at all possible. In the initial or experimental stages of constructing patterns and designs, paper or cardboard pattern blocks are a poor substitute for the real plastic or wooden ones. Paper pattern blocks are very useful, however, for recording patterns made with the real blocks. For a supply of paper pattern blocks, run off the following blackline masters on coloured construction paper:

BLM II-5, *yellow* hexagons
BLM II-6, *red* trapezoids
BLM II-7, *orange* squares
BLM II-8, *blue* rhombuses
BLM II-9, *green* triangles
BLM II-10, *grey* rhombuses

Cut off the sides of cardboard cartons to use as trays for pattern-block constructions. Students can store unfinished designs on a tray to finish at a later time.

2-D shapes See the *Materials* section of Learning Opportunity I (page 16).

COMMENTS ON ACTIVITIES

LO II-1 The set of 3-D shapes used for this activity must include building blocks. You may wish to assign just steps 1, 2, and 3 of this card to younger students. Older students should complete all the tasks on the card.

LO II-2 Students will need either 1 cm squared paper (BLM II-1) or 2 cm squared paper (BLM II-2), depending on the size of the interlocking cubes they are using.

LO II-3 Observe the students closely in the initial stages of working with the cubes. They may need assistance with the idea of what constitutes a rectangular prism (1 [b]). Only one arrangement is possible for 2, 3, 5, 7, 11, 13, and 17 interlocking cubes. The 2 x 2 x 2 and the 3 x 3 x 3 arrangements are cubes. Some students may wish to extend this activity by using a greater number of cubes. Encourage them to estimate how many different arrangements can be made for each number of interlocking cubes used and to check their estimate.

LO II-4 Each student will need a copy of BLM II-3. Run this off on construction paper or thin cardboard if possible. Have students work in pairs so that one student can hold the tiles while the other traces around them.

LO II-5 Each student will need the seven tangram pieces (BLM II-4). See the *Materials* section of this learning opportunity for details.

LO II-6 Students will need paper pattern blocks and cardboard for this activity. See the *Materials* section of this learning opportunity for details. Before assigning this activity, allow time for students with little or no experience of working with pattern blocks to engage in free and unguided exploration.

LO II-7 Have students work in groups of two to four. Each group will need a cube, and each player will need a copy of the gameboard (BLM II-11). Have students mark the faces of the cube as shown in the *You need* section of this activity sheet.

LO II-8 Students will need paper pattern blocks. See the *Materials* section of this learning opportunity for details.

The following numbers of different types of blocks will fit completely around a point:

- hexagons — 3
- squares — 4
- trapezoids or blue rhombuses — 3, 4, 5, or 6
- triangles — 6
- grey rhombuses — 4, 8, or 12

Have a set of pattern blocks available for demonstration during the follow-up discussion of this activity.

LO II-9 Each student will need a copy of BLMs II-12 and II-13. Have the students work in pairs so that one student can hold the blocks in place while the other traces.

LO II-10 Have the students work in groups of three or four. Encourage the students to cut carefully so that the tiles will fit leaving little space between them. Students should find that all triangles and all quadrilaterals will tile.

LO II-11 Students will need coloured construction paper. Only the hexagon will tile. The pentagon leaves a triangular gap, and the octagon leaves a square one. Accept the students' explanations of why some shapes tile and others will not. Do not force a theoretical explanation on them.

LO II-12 Students will need the tiles on BLM II-14. In a group follow-up session encourage each student to explain how he or she made the tile used in the tiling pattern and how the tiling pattern was constructed.

Learning Opportunity III

- DEVELOP CONCEPTS OF SYMMETRY RELATED TO POINT, LINE, PLANE, ROTATIONAL, AND TRANSLATIONAL SYMMETRY.

BACKGROUND NOTES

Symmetry plays an important role in many aspects of mathematics, as well as in areas such as art, music, literature, and architecture. Activities involving paper folding and cutting, rotating of figures about a turn centre, and examining images by means of a Mira will help students build up an intuitive understanding of the concept of point and line symmetry. Slicing Plasticine models of three-dimensional figures along what is thought to be a plane of symmetry and checking the half figure on a mirror will help students grasp the idea of plane symmetry. Making drawings, tracings, or collages of designs and geometric patterns observed in nature (for example, the arrangement of the petals on a flower or the patterns on a snake skin, turtle shell, or sea shell) will give students insight into the properties of symmetry. Students can explore properties of rotational symmetry by sticking a pin through a turn centre in, on, or outside a two-dimensional figure and physically rotating it.

Students may be introduced to the concepts involved in translational symmetry by examining and producing strip patterns and overall designs, such as those on wallpaper and wallpaper borders.

MATERIALS

Check the individual activity sheets for the complete list of materials for each activity.

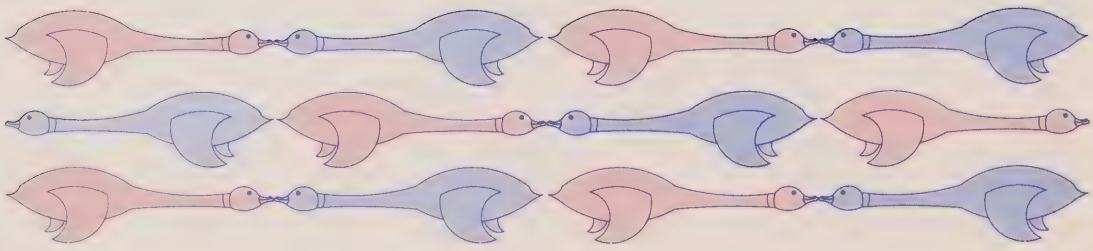
Mira This red plexiglass aid for discovering geometrical properties, including symmetrical relations, and for drawing flip images is a highly recommended tool for Junior Division geometry. You may purchase it from most educational suppliers.

Onion-skin paper This paper is useful for activities that involve tracing, folding, and cutting.

COMMENTS ON ACTIVITIES

LO III-1 Initiate a group discussion of the paint-blob pictures after the students have completed step 4 of this activity. Students may want you to be part of the planning described in 5(a). Have younger students tape record their stories of what they imagine is happening in the mural. Have older or more mature students write and publish their stories.

LO III-2 Students will need thin paper that folds readily and cuts easily even through four thicknesses. Have onion-skin paper available.



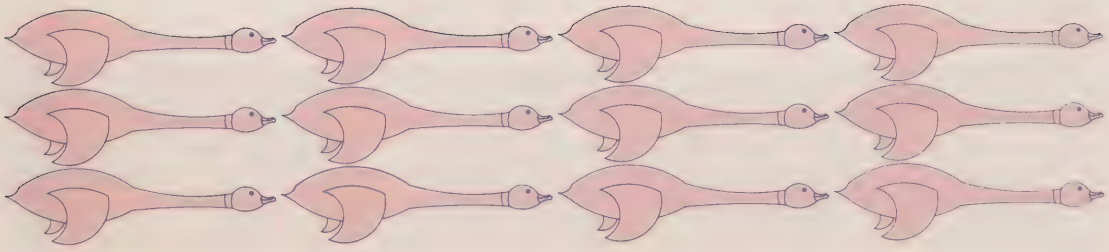
LO III-3 Students will need BLM III-1 and a Mira for this activity. An ordinary mirror is a poor substitute. See the *Materials* section of this learning opportunity for details. Allow time for students who have never used a Mira to explore with it in order to develop an appreciation of its usefulness. Make sure that students are able to see images of objects on one side of the Mira, to draw the images, and to use the Mira's bevelled edge to draw straight lines, before assigning this activity. Have students work in pairs.

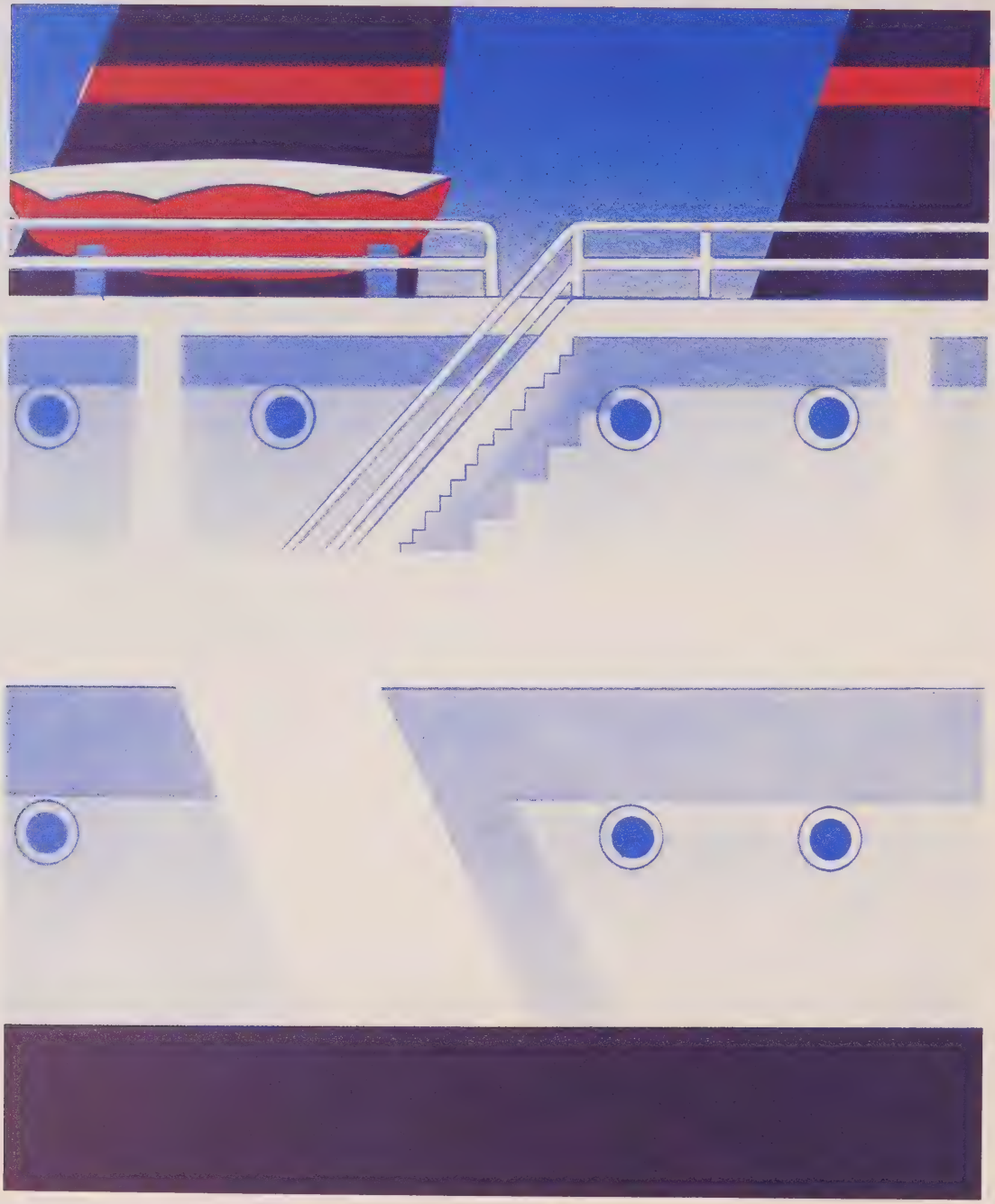
LO III-4 Students will need an ordinary mirror.

You need not require students to find the exact number of planes of symmetry of each 3-D model.

LO III-5 The students will need 1 cm squared paper (from BLM II-1). Take the class to visit a zoo to study patterns in nature. If that is not possible, encourage the students to look in books and encyclopaedias for information on such things as spiders and spiders' webs, snakes and snake skins, the architecture of beehives and honeycombs, snail shells, sea shells, birds' nests, animal tunnels, and anthills. Enlist the help of your school librarian.

LO III-6 Make sure that all members of each group share in the preparation of this game.





Learning Opportunity IV

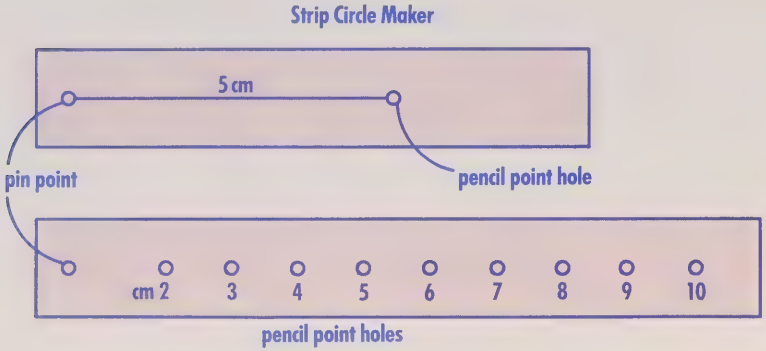
- IDENTIFY THE SYMMETRIES OF FIGURES IN THE PLANE AND OF OBJECTS IN SPACE

BACKGROUND NOTES

Very young children display an intuitive understanding of symmetry through their painting and drawing. They can appreciate the symmetry of a butterfly long before the concepts of distance or number have any meaning. In the Primary Division, children acquire experience of working with geometric models and shapes, making two-dimensional patterns and designs, reflecting objects in a mirror or Mira, and constructing symmetrical figures with paint blobs on folded paper or by cutting folded paper. Learning Opportunity II and Learning Opportunity III provide experiences with symmetries of tiled and stacked patterns, with lines and planes of symmetry, and with rotational and translational symmetry. The activities in Learning Opportunity IV are directed specifically towards helping students identify symmetries of two-dimensional and three-dimensional shapes.

Check the individual activity sheets for the complete list of materials for each activity.

Compasses Older or more mature students in the Junior Division may have developed the fine motor skills that allow them to use regular compasses with ease and enjoyment. Provide cardboard-strip circle makers for those students who have not yet developed to the point where they can use regular compasses without becoming frustrated. Construct circle makers from strips of cardboard. You may use separate strips for each radius needed, or combine all the radii on one strip.



Punch a hole at each end of the radius, a small one at one end for a pin or thumbtack point, and a slightly larger one at the other for a pencil point. With one hand students hold the pin or thumbtack in place at the turn centre, and with the other they draw a circle by grasping the pencil and swinging the strip in a complete turn around the centre. With a bit of practice, all students learn to use this circle maker with ease. The advantage of this tool is that the length of radius is more flexible than with regular compasses.

3-D models and 2-D shapes See the *Materials* section in Learning Opportunity I (pages 15-16) for details.

Mira See the *Materials* section in Learning Opportunity III (page 26) for details.

COMMENTS ON ACTIVITIES

LO IV-1 Students will need a cup of sand or salt, regular compasses or cardboard-strip circle makers, and some tape. See the *Materials* section in this learning opportunity for information on cardboard-strip circle makers.

Have the students work in pairs to help each other tape the circles to make cones.

LO IV-2 Students will need Plasticine, drinking straws, and (optionally) pipe cleaners. Have younger students do steps 1, 2, and 3 of this activity. In step 4, you may want to limit the use of pipe cleaners to the more mature students.

LO IV-3 Students will need copies of BLM IV-1. Have students work in pairs.

The number of fold lines for each of the figures on BLM IV-1 are as follows:

- equilateral triangle — 3
- isosceles triangle — 1
- square — 4
- rectangle — 2
- parallelogram — 0
- regular pentagon — 5
- regular hexagon — 6
- circle — countless

LO IV-4 Purchase butterfly paper fasteners (available from most office-supply or variety stores). Use the sharp metal point of a compass if no paper punch is available. Have students work in pairs.

LO IV-5 Students will need regular compasses or cardboard-strip circle makers. See the *Materials* section in this learning opportunity for details. Encourage the students who construct a rosebud design to colour congruent parts the same colour.

LO IV-6 Students will need copies of the 2-D patterns for 3-D models from BLMs IV-2 through IV-10. They will also need 2 cm squared paper (from BLM II-2). Have students work in groups of three or four, so they can help each other.

Learning Opportunity V

- UNDERSTAND AND USE WORDS SUCH AS *SLIDE, TURN,* AND *FLIP* IN DESCRIBING MOTIONS OF FIGURES IN THE PLANE AND IN SPACE

BACKGROUND NOTES

In 1872 a famous mathematician, Felix Klein, said that all geometry could be thought about by moving or changing a geometric figure and observing which properties do *not* alter. When we slide, flip, or turn a figure, only the position changes. Such attributes as length, width, area, and angle size remain the same. Students' early experiences with motion geometry will give them a firm foundation for the study in later years of transformational geometry — translations (slides), reflections (flips), and rotations (turns).

In many of the activities in Learning Opportunities II, III, and IV, students used the terms *slide, flip,* and *turn,* to describe the motions used to make strip patterns, tiling patterns, and symmetrical figures and designs. Take advantage of every opportunity in the classroom, in the gym, and outside in the playground to use these terms to describe motions. For example, ask the students to *slide* to the right, to the left, forward, and backward without changing the direction they are facing. Ask them to make a one-quarter *turn,* a half turn, a three-quarter turn, and a whole or complete turn, both clockwise and counter-clockwise. Ask them to *slide, turn,* and *flip* objects such as a box, a book, or a shoe, and to describe the appearance of the object after each motion.

MATERIALS

Check the individual activity sheets for the complete list of materials for each activity.

Geoboards See the *Materials* section in Learning Opportunity I (page 16) for details.

Mira See the *Materials* section in Learning Opportunity III (page 26) for details.

3-D models and 2-D shapes See the *Materials* section in Learning Opportunity I (pages 15-16) for details.

COMMENTS ON ACTIVITIES

LO V-1 Students will need long strips of paper (adding-machine tape is excellent) for making strip patterns. Encourage the students to examine strip patterns made by other students and to describe the slide motion and the way colour adds to the pattern.

LO V-2 Students will need dot paper to match the geoboards (from BLM I-5 or VII-1).

Place a removable sticker on one corner of the geoboard to make it easier for students to follow the turns. Have students work in groups of three or four.

LO V-3 Students will need dot paper (BLM I-5 or VII-1) and a geoboard, preferably the 10 x 10 model. A triangular board would be satisfactory, but a 5 x 5 board might be too small.

LO V-4 Students will need square dot paper (BLM I-5).

LO V-5 The strip patterns shown in step 3 are made by two motions — a slide and a turn. If you use a turn centre to move the shapes, the centre would lie outside the shapes. Discuss this with the students if someone asks about it. If not, be satisfied with the experiences the students will have seeing how shapes appear after a one-half or a one-quarter turn and a slide.

LO V-6 Make sure that the students are able to use a Mira to draw flip images before assigning this activity.

LO V-7 Have students draw diagonals to find the centre point of the square, the hexagon, and the rectangle. Some students may need to make a paper cutout of the equilateral triangle and find the centre by folding.

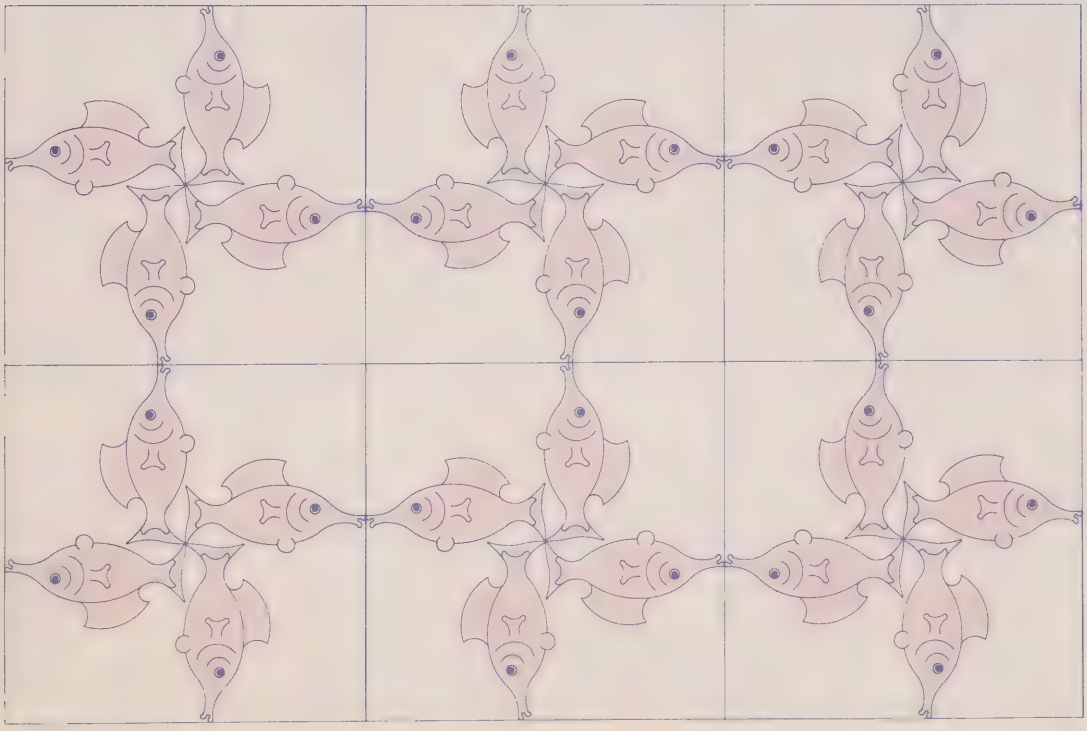
LO V-8 Students will need onion-skin paper and carbon paper. Ask the students to bring their image pictures to the follow-up discussion and use them in describing their experiences.

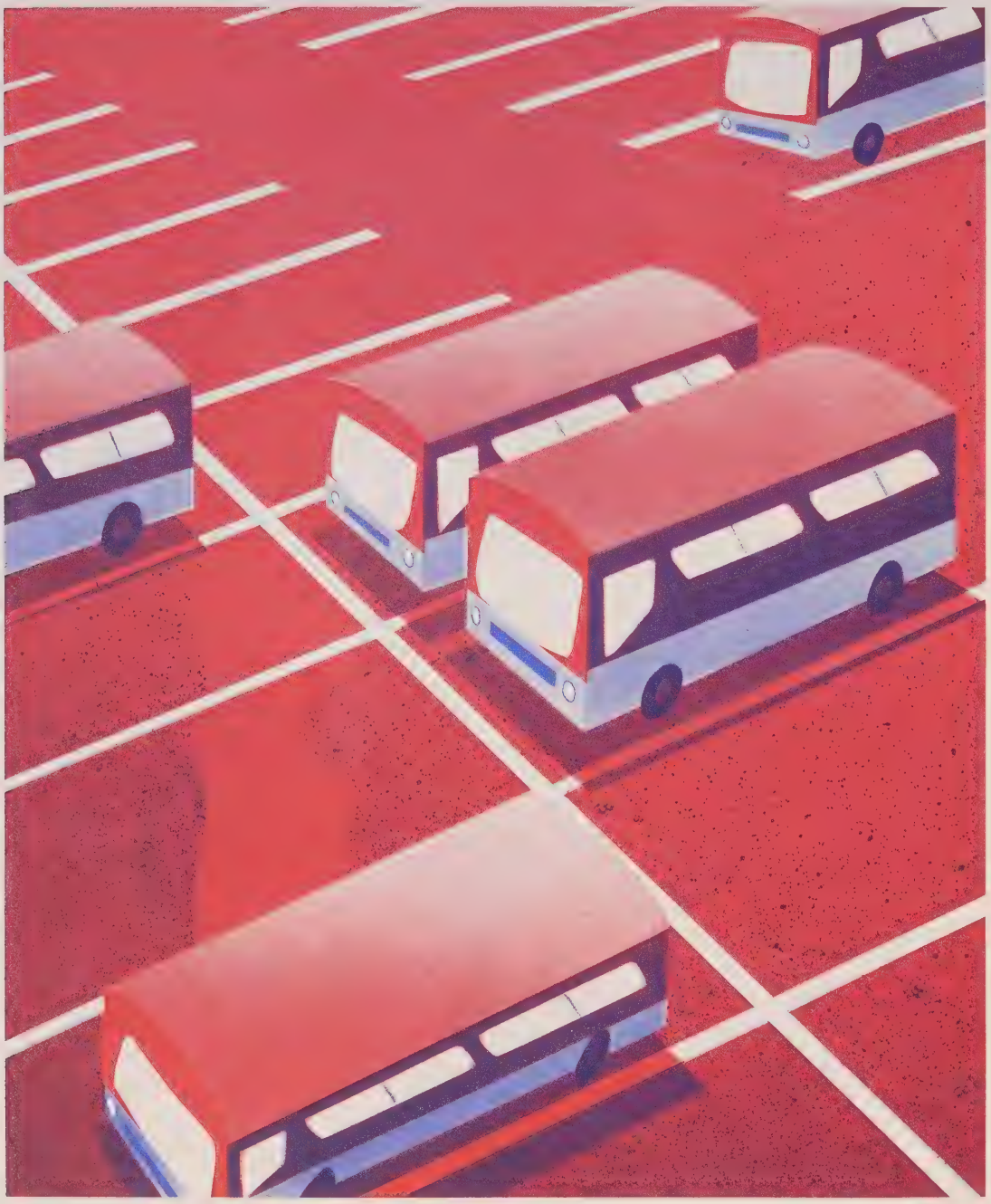
LO V-9 Students choosing a turn centre outside a shape will have to solve the problem of how to turn the shape about this turn centre. Encourage the students to experiment and to share ideas as they determine a solution.

LO V-10 Students will need large square and rectangular pieces of paper. Let students choose the size of the sections referred to in step 1(a) of this activity.

LO V-11 Students will need 1 cm squared paper (BLM II-1). Encourage the students to make the tiles they like best from 1(a), (b), and (c).

LO V-12 The chart to be developed in this activity is intended as an aid to the students. When completed, it will contain a summary of what they have learned about slide, flip, and turn motions. Use step 4 as the basis of a follow-up class discussion.





Learning Opportunity VI

- DEVELOP AND USE THE CONCEPT OF CO-ORDINATES

BACKGROUND NOTES

Early experiences on a jungle gym in the playground and on the climbing bars in the gymnasium help children to understand movement in space — up and down, left and right, backwards and forwards, inwards and outwards. A number plane is the union of a horizontal number line and a vertical number line. The point of intersection of these two lines is labelled *O*. The number on the horizontal line is always named first, and the number on the vertical line is named last. The grid lines help to locate the numbers on the horizontal and vertical lines. If a point is above the 4 on the horizontal number line, and to the right of the 7 on the vertical number line, then that point is located at (4,7); four and seven are the *co-ordinates* for that point. The first quadrant only is used in the activities for Junior Division students. In the Intermediate Division students will use all four quadrants, involving positive and negative numbers.

MATERIALS

Check the individual activity sheets for a complete list of materials for each activity.

Cubes Two-centimetre plastic or wooden cubes make very good dice. Write directly on the faces, or use small self-adhesive labels.

Number planes Three number planes containing the point (5,5) and two number planes containing the point (10,10) are provided on BLM VI-1. BLM VI-2 is the gameboard for the game of Co-ordinates. BLM VI-3 provides one number plane containing the point (20,10) and one number plane containing the point (15,15).

COMMENTS ON ACTIVITIES

LO VI-1 Students will need a number plane containing the point (5,5). See BLM VI-1. When students' desks or chairs are arranged in rows, take advantage of the opportunity to talk about row numbers and seat numbers. Have the students draw seating plans, picturing rows and seats (see seating plan LO VI-1). When the points in step 4 are joined, the image of sailboat is produced.

LO VI-2 For the game of Co-ordinates, students will need the gameboard on BLM VI-2 and twenty pieces of 1 cm x 1 cm cardboard per player. (The game may be played by two to four players.)

LO VI-3 Each player will need a copy of a number plane containing the point (10,10). See BLM VI-1. You may wish to introduce this game to the class by acting as captain in one or two games of Search and Rescue.

LO VI-4 Each student will need a (20,10) number plane. See BLM VI-3.

LO VI-5 Each student will need at least four (10,10) number planes and at least two (15,15) number planes. See BLMs VI-1 and VI-3.

LO VI-6 Each student will need three number planes containing the point (15,15). See BLM VI-3.

Learning Opportunity VII

- UNDERSTAND, REPRESENT, AND USE CONCEPTS SUCH AS *VERTICAL, HORIZONTAL, PERPENDICULAR, PARALLEL, ANGLE, TRIANGLE,* AND *POLYGON*

BACKGROUND NOTES

Through play activities with building blocks, Primary children acquire an informal, intuitive awareness of the concepts identified in this learning opportunity. Through sorting and classifying activities children in the Primary Division learn to identify triangles, squares, rectangles, and other polygons.

In the Junior Division, learning experiences with tiling patterns and symmetry extend the understanding of the concepts of vertical, horizontal, perpendicular, parallel, and angle, and of angle measurement. Through sorting and classifying activities, students begin to develop the ability to classify polygons according to families, recognizing attributes common to all family members.

The exploration activities in Learning Opportunity VII are structured so that experiences with these important geometric concepts are not left to chance. However, do not lose sight of the fact that students in the Junior Division still need to be free to explore concepts at an intuitive level and to express observed relationships *in their own words.*

MATERIALS

Check the individual activity sheets for a complete list of the materials for each activity.

Geoboards, 3-D models, 2-D shapes See the *Materials* section of Learning Opportunity I (pages 15-16) for details.

COMMENTS ON ACTIVITIES

LO VII-1 Students will need dot paper to match the geoboards (BLM I-5 or VII-1).

LO VII-2 Students will need protractors, chart paper, and a set of 2-D shapes. Set B will have no members that are also in Set A or Set C.

LO VII-3 Students will need dot paper to match the geoboards (BLM I-5 or VII-1). Triangles are impossible with two or three right angles, and two or three obtuse angles. Quadrilaterals are impossible with four acute angles.

LO VII-4 Students will need a clear plastic jar, coloured water, string, and a small mass such as a nut, bolt, or fishline sinker.

LO VII-5 Square ruled paper could be described as having sets of vertical and horizontal parallel lines perpendicular to each other.

LO VII-6 Students will need both square and triangular geoboards and dot paper to match each (BLMs I-5 and VII-1). Have students work in pairs.

Learning Opportunity VIII

- UNDERSTAND THE CONCEPT OF CONGRUENCY

BACKGROUND NOTES

Figures are congruent if they have the same shape and size. They can be tested for congruency by tracing, sliding, turning, and flipping to fit one on the other, or by measuring.

Primary Division children acquire an intuitive and working knowledge of the concept of congruency through activities involving building blocks, pattern blocks, strip and all-over patterns, and other symmetrical designs.

Junior Division students need to deepen and extend this knowledge through activities involving sorting and classifying, measuring, tiling and stacking, and slide, flip, and turn motions. Take advantage of every opportunity for the students to check for congruency by tracing and fitting or by measuring what appear to be corresponding sides and angles of polygons. In the construction of tiling patterns and line and circle designs, encourage the students to colour congruent parts the same colour.

MATERIALS

Check the individual activity sheets for a complete list of the materials for each activity.

3-D models and geoboards See the *Materials* section of Learning Opportunity I (pages 15-16) for details.

COMMENTS ON ACTIVITIES

LO VIII-1 Students will need chart paper and a set of 3-D models.

LO VIII-2 Have the students draw shapes on cardboard and cut them out.

LO VIII-3 Students will need geoboards and dot paper to match (BLM I-5 or VII-1). Have students work in groups of two to four.

LO VIII-4 Each student will need a copy of BLM VIII-1. This is one way to form an octagon.

LO VIII-5 Have students work in pairs. Each pair of students will need two copies of BLM VIII-2. These should be in different colours.

LO VIII-6 Each group of students will need a copy of the twelve pentominoes and one tetromino (BLM VIII-3) and a sheet of 2 cm squared paper (BLM II-2). If possible, laminate the pentominoes, the tetromino, and the 8 x 8 (64 squares) gameboard. Have students work in groups of three or four.

Learning Opportunity IX

- IDENTIFY RELATIONSHIPS BETWEEN SIMILAR FIGURES AND APPLY MEASUREMENT SKILLS IN SCALE DRAWINGS

BACKGROUND NOTES

By the time most students reach the Junior Division, they will have some intuitive understanding of relationships between similar figures through experiences with such things as scale models of real objects, different sizes of clothing, television screens, photographs and enlargements, magazine and billboard advertisements, etc.

In the Primary Division, children will have constructed three-dimensional and two-dimensional maps of the classroom, the school, and probably of the neighbourhood or community. Although these maps were not likely constructed to scale, the shapes and distances would have been roughly preserved. They may have developed some understanding of relationships such as twice as big, half as high, and so on.

Junior students need to extend these concepts to use their measurement skills to explore, mathematically, relationships between similar figures and to apply this knowledge in making scale drawings.

MATERIALS

Check the individual activity sheets for a complete list of the materials for each activity.

Geoboards See the *Materials* section of Learning Opportunity I (page 16) for details.

Pattern blocks See the *Materials* section of Learning Opportunity II (page 22) for details.

COMMENTS ON ACTIVITIES

LO IX-1 Students will need square dot paper and triangular dot paper (BLMs I-5 and VII-1).

Squares		Triangles	
Number of units on sides	Total number of unit squares	Number of units on base	Total number of unit triangles
1	1	1	1
2	4	2	4
3	9	3	9
4	16	4	16

Some students will see that the pattern is the same for both squares and triangles, and may also see a relationship between the length, in units, of the sides or base and the total number of unit tiles. A few students may be able to tell how many unit squares or triangles would be in a square or triangle with sides or a base of eight, nine, ten, or more.

LO IX-2 Students will need three sizes of squared paper for the scale drawings. See BLMs II-2 (2 cm squares), II-1 (1 cm squares), and IX-1 (0.5 cm squares).

LO IX-3 Students will need the three sizes of squared paper used in LO IX-2 and a large sheet of paper (45 cm x 60 cm).

LO IX-4 1 cm squared paper (BLM II-1) is needed. Draw the students' attention to the fact that the area increases more than the perimeter when you make the sides of a rectangle two or three times longer.

LO IX-5 Students will need 1 cm dot paper (BLM I-5 or VII-1), plus similar figures (BLM IX-2). If they start to copy the designs of squares onto dot paper, starting at the centre in 1(d) and at the bottom left corner in 2(d), they will find that they can continue both patterns for four more squares.

LO IX-6 The students will need a map of the community or an official road map of Ontario (see the *Materials* section of Learning Opportunity I, page 17). They will also need 1 cm squared paper (BLM II-1).

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